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Research Paper

Improvement of the Focusing Inversion of Gravity Data with Hybrid Conjugate Gradient Parameter Method

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Abstract: Potential field methods, such as the gravity technique, have become essential tools for exploration. Inversion of gravity data is the most important stage in the interpretation of the data. Inversion is a mathematical technique that constructs a geophysical subsurface model automatically from measured data by adding some prior knowledge. Inversion of gravity data is time-consuming and needs a long time because of numerous data and model parameters. Thus, a fast and effective inversion method is necessary to improve the speed of the inversion process. Many algorithms are available for focusing inversion of gravity data, such as the reweighted regularized conjugate gradient (RRCG) method. This method is iterative, and it takes a long time to converge to a solution. In this algorithm, there is a conjugate gradient parameter that is effective in inversion. In this paper, we used a hybrid conjugate gradient parameter method for focusing inversion of gravity data and compared the results with the conventional Fletcher-Reeves (FR) conjugate gradient parameter method. We applied this method for the data from a synthetic model and Shoaz iron ore deposit in Yazd, Iran. The inversion result indicated that the hybrid conjugate gradient parameter method converges to the solution faster than the FR method, while the results from both approaches have remarkable correlations with the true geological structures.

Keywords: Inverse modeling, RRCG algorithm, Gravity, Conjugate gradient parameter, Shoaz ore deposit.

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INTRODUCTION

Inversion of potential field data is the most critical stage in the quantitative interpretation of the data. The solution of the inverse problem is unstable and non-unique. The issues can be overcome by the Tikhonov regularization approach [1]. In this approach, the prior information is incorporated in the model by a stabilizing function (stabilizer) [2-4]. Some smooth stabilizers produce smooth models with fuzzy borders, while the focusing stabilizers reconstruct piecewise constant models with sharp boundaries [2].

On the other hand, Inversion of gravity data is time-consuming and needs a long time because of numerous data and model parameters. Thus, a fast and effective inversion method is necessary to improve the speed of inversion [5]. Many algorithms are available for focusing inversion of gravity data, such as the reweighted regularized conjugate gradient (RRCG) method. This method is iterative, and it takes a long time to converge to the solution [6]. In this algorithm, there is a conjugate gradient parameter that is effective in the number of iterations during the inversion process. This conjugate gradient parameter is calculated by the Fletcher-Reeves (FR) method [7]. In this paper, we used a hybrid conjugate gradient parameter method for focusing inversion of gravity data which is a combination of FR and PRP techniques. This method converges to the solution faster than the conventional FR method while both techniques produce similar solutions. The effectiveness of the method was evaluated by the data from a synthetic model with three blocks. The introduced technique was also applied to the real data from the Shoaz iron ore deposit in Yazd.

METHODS

The aim of the inversion is to use the measured gravity response data to recover the subsurface rock density contrast. An acceptable model is the one that makes a sufficiently small misfit. The regularization functional incorporates information about the basic properties of the type of models used in the inversion [8]. In this paper, we used the RRCG algorithm for focusing inversion of gravity data. In this algorithm, the step size and search direction are essential. In each iteration, a line search method computes a search direction and a step size and decides how far to move along this direction. The conjugate gradient parameter influences the search direction. The FR method is the conventional method for the conjugate gradient parameter [7]:

$$\beta_k^{FR} = \frac{\left\| \mathbf{I}^{\alpha}_{(\mathbf{m}_{n+1})} \right\|^2}{\left\| \mathbf{I}^{\alpha}_{(\mathbf{m}_{n})} \right\|^2} \tag{1}$$

Where:

$\mathbf{I}^{\alpha}_{(\mathbf{m}_{n})}$: represents the gradient direction of the Tikhonov objective function in the n^{th} iteration.

The FR : is characterized by a strong global convergence rate, but it is not computationally powerful due to the jamming phenomenon. It may take infinitely many steps without reaching the optimum.

Some methods such as PRP may not always converge, but they are often more efficient, computationally [9-13]:

$$\beta_k^{PRP} = \frac{\mathbf{I}^{\alpha T}_{(\mathbf{m}_{n+1})} \left(\mathbf{I}^{\alpha}_{(\mathbf{m}_{n+1})} - \mathbf{I}^{\alpha}_{(\mathbf{m}_{n})} \right)}{\left\| \mathbf{I}^{\alpha}_{(\mathbf{m}_{n})} \right\|^2} \tag{2}$$

Hybridization is one of the popular approaches in modifying the conjugate gradient method. We have used the hybrid method as follows:

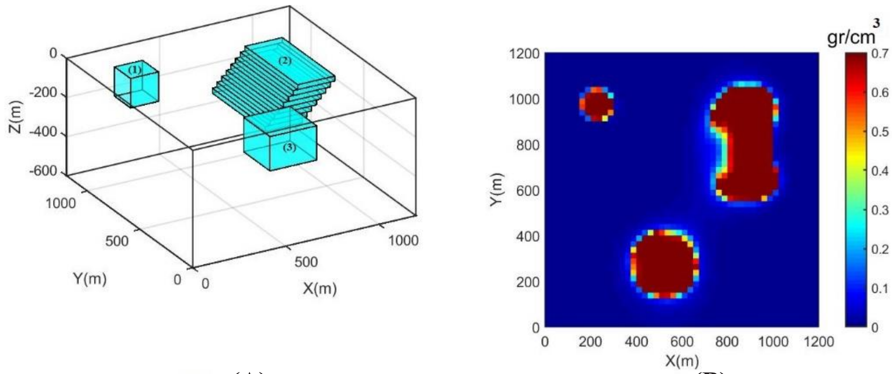
$$\beta_k = \max \left\{ \beta_k^{FR}, \beta_k^{PRP} \right\} \tag{3}$$

Combining these classical methods in Equation 3 will allow us achieving a global convergence for the inverse problem with a better computational performance.

SYNTHETIC AND REAL EXAMPLES

The proposed method has been applied to synthetic data from a synthetic model with three blocks

(Figure 1A). The results indicate the new method has produced a focused solution (Figure 1B). One can see the new method is faster than the traditional approach (Table 1).



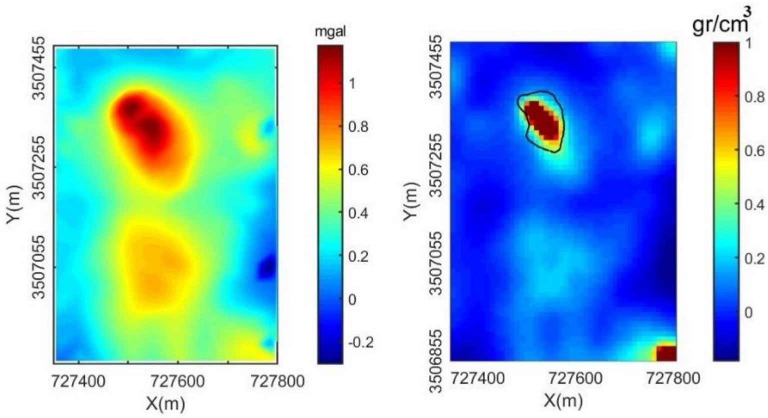
(A) **(B)**

Figure 1. A: 3D view of the synthetic model, **B:** Plan section through the recovered model at the depth of 100 m

Table 1. Times and iteration model

Model	Conjugate gradient parameter	Iteration	Time (s)
Synthetic	FR	1984	1929
	Hybrid	1865	1819
Shoaz	FR	55	11
	Hybrid	48	5/59

We have also applied the proposed method for inversion of gravity data over the Shoaz iron deposit to show the reliability of the new approach for gravity inversion. The results indicate that the new technique converges to the solution faster than the traditional method, while the results are geologically plausible (Figure 2).



(A) **(B)**

Figure 2. A: Residual gravity data over Shoaz Iron deposit, B: The plan section through the recovered model 40 m below surface and the black line is the border of the deposit

CONCLUSIONS

The inversion of gravity data is significant for the determination of the distribution of subsurface densities. The focusing inversion approach is a robust method for inversion of gravity data. The inversion process is slow. Thus, we have used a hybrid method for computing conjugate gradient parameter that combines the FR and PRP methods. The inversion results of synthetic and real data from the Shoaz iron deposit show that the hybrid technique is faster than the FR method to converge the solution. However, the models from both methods have good similarities with the actual structures.

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